Math 158 – Linear Models, Spring 2022 Leverage Points and Residuals

Statistic	Formula	Extreme?	R
Leverage	$h_i = \frac{(X_i - \overline{X})^2}{\sum_{j=1}^n (X_j - \overline{X})^2} + \frac{1}{n} = \mathbf{X}_i^t (\mathbf{X}^t \mathbf{X})^{-1} \mathbf{X}_i$	$> \frac{2p}{n}$ or .25 = moderate, > .5 high	.hat
DFFITS	$\frac{\hat{Y}_i - \hat{Y}_i(i)}{\sqrt{MSE_i - b_i}}$	> 1 for med-sized data sets,	dffits()
	$\bigvee^{III \cup L_{(i)} n_{ii}}$	$> 2\sqrt{\frac{p}{n}}$ for large data sets	
Cook's Distance	$D_{i} = \frac{\sum_{j=1}^{n} (\hat{Y}_{j} - \hat{Y}_{j(i)})^{2}}{pMSE}$	≥ 1	.cooksd
DFBETAS	$\frac{b_k - b_{k(i)}}{\sqrt{MSE(\gamma)Chk}}$	> 1 for med-sized data sets,	dfbetas()
	$c_{kk} = (\mathbf{X}^t \mathbf{X})_{kk}^{-1}$	$> 2/\sqrt{n}$ for large data sets	
Resid	$e_i = (Y_i - \hat{Y}_i)$.resid
Semi-studentized Resid	$rac{e_i}{\sqrt{MSE}}$	outside (-2,2)	.std.resid
(Internally) Studentized Resid	$\frac{e_i}{\sqrt{MSE}\sqrt{1-h_{ii}}}$	outside (-2,2)	
Deleted Studentized Resid	$\frac{e_i}{\sqrt{MSE_{(i)}}\sqrt{1-h_{ii}}}$	outside $(-2,2)$	rstudent()
VIF	$(1-R_k^2)^{-1}$	$\max(VIF) > 10$	vif()
(for $k^{\prime\prime\prime}$ var, not obs)	K_k^2 from X_k regressed on $(p-2)$ vars	$\operatorname{mean}(VIF) >> 1$	package car

Notes:

- The first four statistics are measures of how **influential** the value is. Leverage measures the distance of the explanatory variables from the average. Cook's distances, and the derivatives, are a measure of how much the predicted values change when the point is removed from the model.
- The residual statistics are measures of how well the regression line fits the value. A residual is the distance from the point to the line. We standardize the residual in different ways. The studentized residuals contain the more accurate measure of standard error.
- The VIF measures the degree of collinearity between the explanatory variables. Collinear variables indicates that we should be cautious interpreting any coefficients. mean(VIF) >> 1 is meant to indicate that the average VIF is considerably larger than 1.
- Any value containing a "(i)" indicates that the i^{th} point was removed before calculating the value. For example, $MSE_{(i)}$ is the MSE for the full model containing all the data **except** the i^{th} point.